Statistics
Homework 4: Continuous Distributions

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1. Solve the following exercises of chapter 4 of Devore, Berk, Carlton's *Modern Mathematical Statistics with Applications* (third edition): 1, 2, 3, 5, 6, 8, 11, 12, 13, 14, 16, 18, 21, 22, 23, 24, 29, 30, 39, 40, 41, 43, 45, 47, 53, 55, 57, 63, 65, 71, 72, 73, 75, 80.

Answers

1. You can find answers to the odd numbered problems at the end of the textbook.

    2. This is an uniform distribution for $-5 < x < 5$ the pdf is $f(x) = 1/10$ and the cdf is $F(x) = (x+5)/10$.

        a. $P(X < 0) = F(0) = 0.5$
        b. $F(2.5) - F(-2.5) = 0.75 - 0.25 = 0.5$
        c. $F(3) - F(-2) = 0.8 - 0.3 = 0.5$
        d. $F(k+4) - F(k) = \frac{k+9}{10} - \frac{k+5}{10} = \frac{4}{10} = 0.4$

    6. The cdf is:

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 2 \\ \frac{3}{4}\left[x - \frac{(x-3)^3}{3} - \frac{7}{3}\right] & \text{if} \quad 2 \le x \le 4 \\ 1 & \text{if} \quad x > 4 \end{cases}$$

        a. See figure.
        b. $k = 3/4$
        c. $1 - F(3) = 1 - 0.5 = 0.5$
        d. $F(3.25) - F(2.75) = 0.6836 - 0.3164 = 0.3672$
        e. $P(X > 3.5) + P(X < 2.5) = 1 - F(3.5) + F(2.5) = 1 - 0.84375 + 0.15625 = 0.3125$

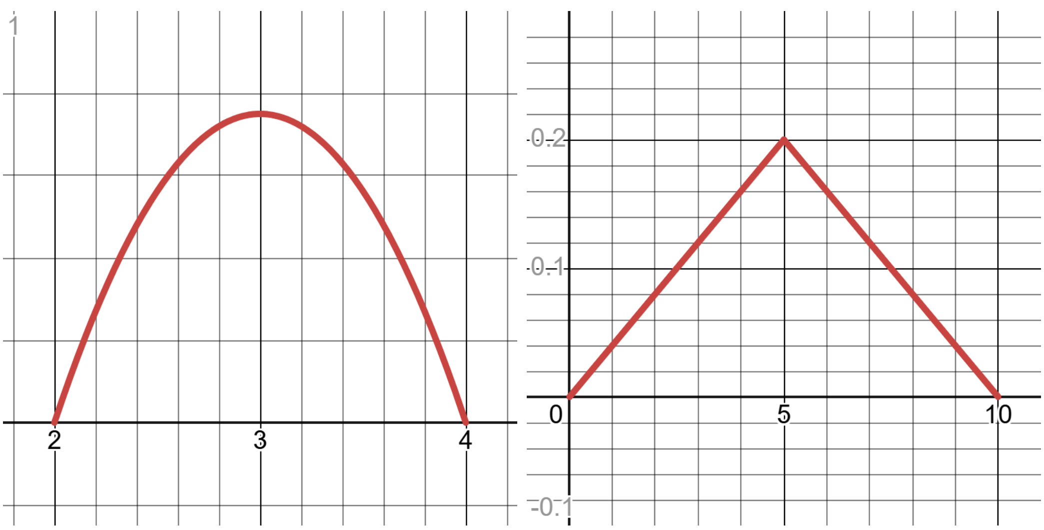

Figure 1: Exercises 6 and 8

    8. Find the cdf by pieces.

$$F(y) = \begin{cases} 0 & \text{if} \quad y < 0 \\ \frac{y^2}{50} & \text{if} \quad 0 \le y \le 5 \\ \frac{2y}{5} - \frac{y^2}{50} - 1 & \text{if} \quad 5 \le y \le 10 \\ 1 & \text{if} \quad y > 10 \end{cases}$$

        a. See figure.
        b. You should verify that $\int_0^5 \frac{t}{25} dt = 0.5$ and $\int_5^{10} \frac{2}{5} - \frac{t}{25} dt = 0.5$
        c. $F(3) = 9/50$

d. $F(8) = 46/50$

e. $F(8) - F(3) = 37/50$

f. $P(X < 2) + P(X > 6) = F(2) + 1 - F(6) = 4/50 + 1 - 34/50 = 20/50$

12. a. 0.5

    b. 0.6875

    c. 0.316406

    d. $f(x) = \frac{3}{32}(a - x^2)$

    e. Set $F(x) = 0.5$ and solve for x.

14. $F(x) = 90\left(\frac{x^9}{9} - \frac{x^{10}}{10}\right)$

    a. See figure.

    b. 0.010742

    c. 0.010713

    d. $x \simeq 0.9$, you don't need to solve exactly for the value.

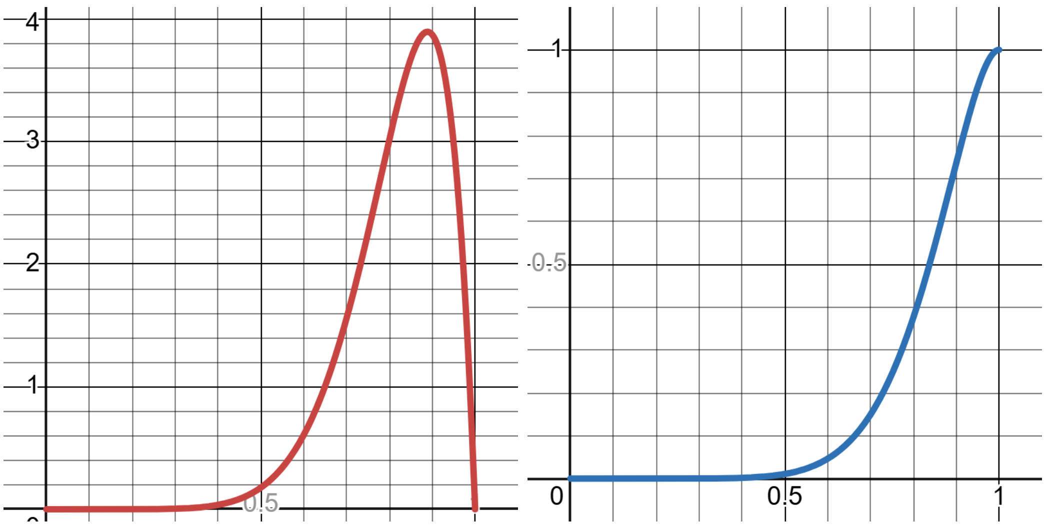

Figure 2: Exercise 14

16. a. $F(1) = 0.596574$

    b. $F(3) - F(1) = 0.369188$

    c. $f(x) = \frac{1}{4}\ln\left(\frac{4}{x}\right)$ for $0 < x < 4$ and zero otherwise.

18. a. 4/3

    b. 2/9 and $\sqrt(2)/3$

    c. 2

22. a. Solved in question 6. See figure.

    b. The median is 3.

    c. $E(X) = 3$, $E(X^2) = 9.2$, and $V(X) = 0.2$

24. a. Solved in question 8. See figure.

    b.

$$y(p) = \begin{cases} \sqrt{50p} & \text{if} \quad 0 \le p \le 0.5 \\ 10 - \sqrt{50(1-p)} & \text{if} \quad 0.5 < p \le 1 \end{cases}$$

    c. $E(X) = 5/3 + 10/3 = 5$, $E(X^2) = 25/4 + 275/12 = 175/6$, $V(X) = 175/6 - (5)^2 = 25/6$. You should calculate the integrals piece by piece.

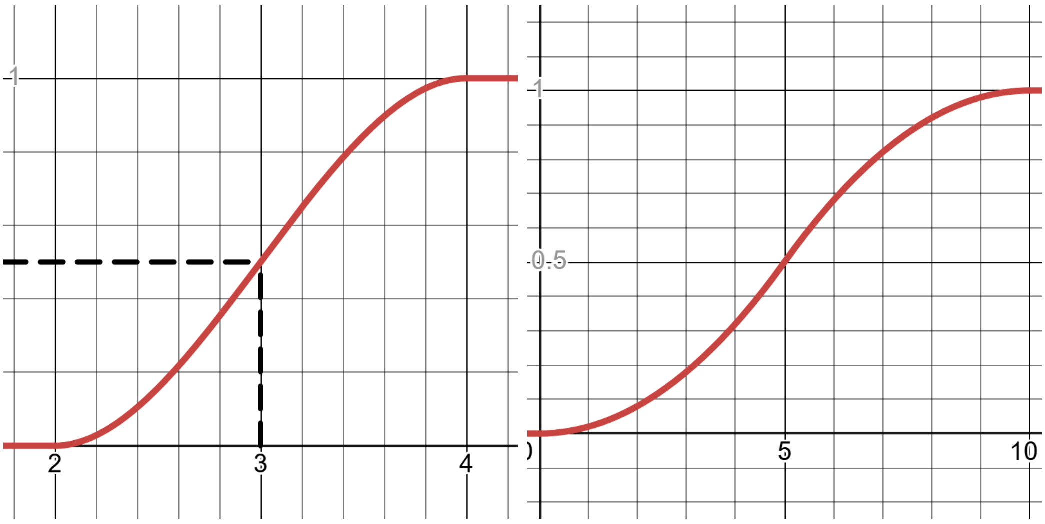

Figure 3: Exercises 22 and 24

30. b. Skewness would be zero. In a symmetric distributions values above the mean will net out with values below the mean.

40. a. 2.13944
    b. 0.8099
    c. 1.17
    d. 0.97009
    e. 2.40892

72. a. 0.23782
    b. 0.23782
    c. 0.31337
    d. $0.68663 - 0.03351$
    e. $0.68663 - 0.03351$
    f. $0.11067 + 0.60630$

80. Its density is $f(x) = \lambda e^{-\lambda x}$. First find its cdf.

$$\begin{aligned} F(x) &= \int_0^x \lambda e^{-\lambda t} dt \\ &= -e^{-\lambda t}\Big|_0^x \\ &= 1 - e^{-\lambda x} \end{aligned}$$

Now set $F(x) = p$ and solve for x.

$$\begin{aligned} F(x) &= p \\ 1 - e^{-\lambda x} &= p \\ 1 - p &= e^{-\lambda x} \\ \ln(1-p) &= -\lambda x \\ -\frac{\ln(1-p)}{\lambda} &= x \end{aligned}$$

To find the median, set $p = 0.5$. As an example, if $\lambda = 1$, then the median is: 0.69315. In general it depends on λ.