Statistics
Problem Set 2: Probability Distributions

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1. **Binomial Distribution.** Let $X \sim \text{Bi}(n, p)$.

   (a) Let $n = 10$. For $p \in \{0.2, 0.5, 0.8\}$ plot the pdf and comment on the symmetry or asymmetry.

   (b) Let $X_i \sim \text{Ber}(p = 0.8)$ iid and define $Y = \sum_{i=1}^{10} X_i$. Simulate draws of Y, plot an histogram and comment on the resemblance to $X \sim \text{Bi}(n = 10, p = 0.8)$.

   (c) For each of the following cases plot the pdf and comment on the resemblance to the Poisson distribution with $\lambda = 2$. You should also plot the pdf of the Poisson distribution.

   - $n = 10$ and $p = 0.2$.
   - $n = 100$ and $p = 0.02$.
   - $n = 1000$ and $p = 0.002$.

2. **Beta Distribution.** Consider $X \sim \text{Beta}(\alpha, \beta)$.

   (a) For $(\alpha_1, \beta_1) = (0.5, 0.5)$, $(\alpha_2, \beta_2) = (1, 1)$, and $(\alpha_3, \beta_3) = (2, 2)$ plot the pdfs in the same graph and calculate the mean and variance of the distribution.

   (b) Choose (α, β) such that the distribution is not symmetric.

   (c) **Uniform Distribution**. Focus on $\alpha = \beta = 1$. Write down its pdf and cdf.

3. **Gamma Distribution.** Consider $X \sim \text{Gamma}(\alpha, \beta)$.

   (a) **Exponential Distribution.** Set $\alpha = 1$.

   - Calculate its mean and variance.
   - Write down the pdf and cdf.
   - Plot the pdf for three different values of β of your choice in the same graph.
   - Let $\beta = 1$, calculate $\mathbb{P}[X > 1]$, $\mathbb{P}[X > 2|X > 1]$, and $\mathbb{P}[X > 3|X > 2]$. This property is called memory-less of the exponential distribution.

   (b) **Chi-Square Distribution**. Set $\alpha = \frac{v}{2}$ and $\beta = \frac{1}{2}$.

   - Calculate its mean and variance.
   - Plot the pdf when $v = 1, 2, 3$ in the same graph.

   (c) Choose three specifications of (α, β) and plot the pdfs in the same graph. Make sure $\alpha \neq 1$ and $\beta \neq \frac{1}{2}$ so this is a general case of the Gamma Distribution.

4. **Normal Approximations.** Let $Z \sim \text{N}(0, 1)$. In many cases the standard Normal distribution can be used to approximate other distributions.

   (a) To the Binomial Distribution. Consider $X \sim \text{Bi}(n = 1000, p = 0.6)$. This approximation works well when n is large.

- Calculate $\mathbb{P}[X > 600]$.
- Approximate the probability above by defining $Z \stackrel{\text{Def}}{=} \frac{X-np}{\sqrt{np(1-p)}}$ and finding $\mathbb{P}\big[Z > \frac{600-np}{\sqrt{np(1-p)}}\big]$.

(b) To the Poisson Distribution. Consider $X \sim \text{Po}(\lambda = 900)$. This approximation works well when λ is large.

- Calculate $\mathbb{P}[X > 950]$.
- Approximate the probability above by defining $Z \stackrel{\text{Def}}{=} \frac{X-\lambda}{\sqrt{\lambda}}$ and finding $\mathbb{P}\big[Z > \frac{950-\lambda}{\sqrt{\lambda}}\big]$.