General Equilibrium - Production

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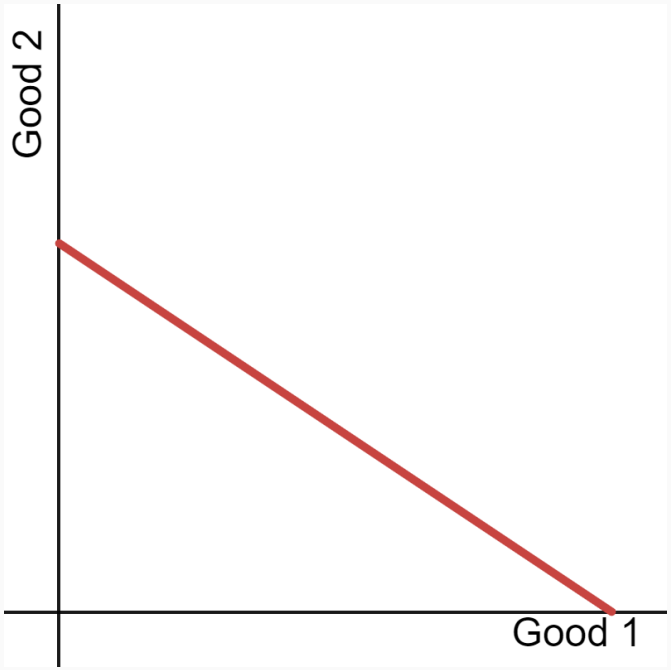
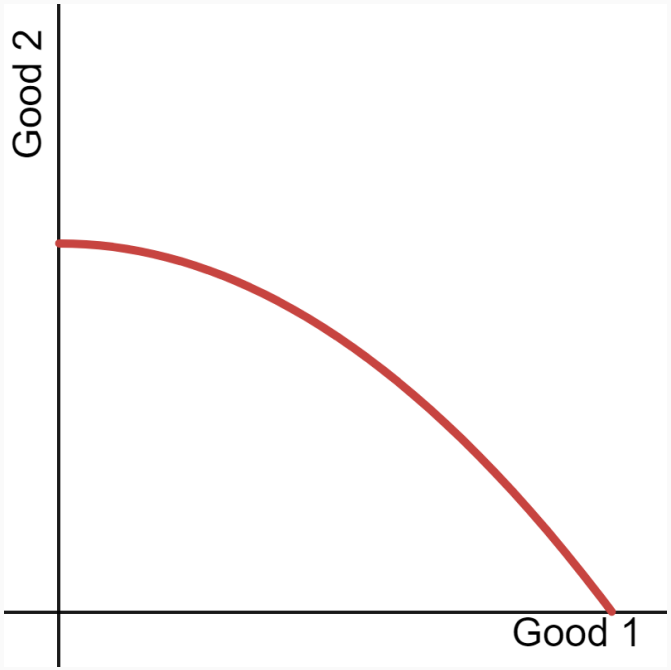
Production Possibilities Frontier

Two Goods, One Factor

- There's a total amount of labor $\bar{L}$ to allocate into the production of goods 1 and 2.
- Production functions: $y_1 = f_1(l)$ and $y_2 = f_2(l)$.
- Let l_1 and l_2 be the labor allocated to the production of goods 1 and 2. Note that $l_1 + l_2 = \bar{L}$.
- All the different choices of (l_1, l_2) define the production possibilities frontier (PPF).
- If $f_1(\cdot)$ and $f_2(\cdot)$ are CRS, then the PPF is linear.
- If one of them is DRS, then the PPF is concave.

Two Goods, One Factor - Graph





MRT

$$\text{MRT} = -\frac{dx_2}{dx_1}$$

- We call the slope of the PPF the Marginal Rate of Transformation between goods 1 and 2.
- How much of good 2 is given up to get an extra unit of good 1.
- It measures the opportunity cost at the margin.
- Changes to the PPF.

Two Goods, Two Factors

- Let $\bar{L}$ and $\bar{K}$ be the total amounts of labor and capital to allocate into the production of goods 1 and 2.
- The production functions are $f_1(k, l)$ and $f_2(k, l)$. We call k_1 and l_1 the amounts of capital and labor used in the production of good 1.
- Extra step: figure out the optimal way of using capital and labor. I want to use inputs efficiently.
- Even if $f_1(\cdot)$ and $f_2(\cdot)$ are CRS, I still can get a concave PPF. See example.

Robinson Crusoe

Environment

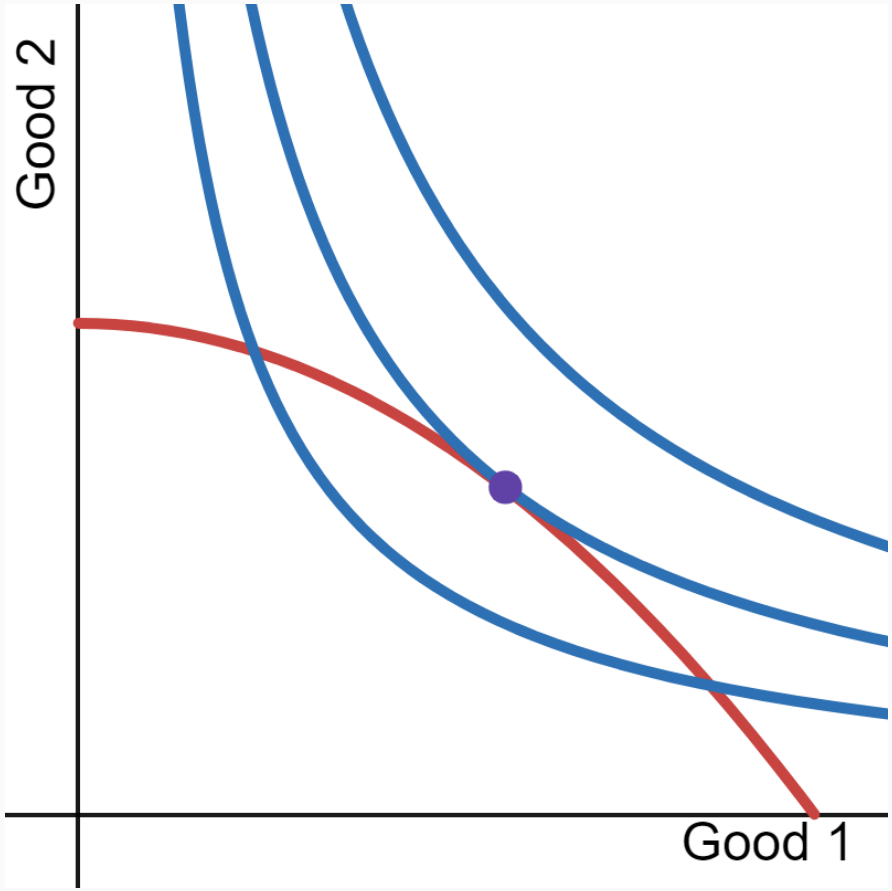
- Robinson Crusoe is alone in an island. His only factor of production is labor. Let $\bar{L}$ be the total time he has to work.
- He can consume fish (good 1) and coconuts (good 2). His preferences are summarized by $u(x_1, x_2)$.
- Note there's no disutility of labor. So he'll work $\bar{L}$ hours. This means $l_1 + l_2 = \bar{L}$.
- $y_1 = f_1(l)$ and $y_2 = f_2(l)$ are the production functions.

Utility Maximization

$$\max_{x_1, x_2} \quad u(x_1, x_2) \quad \text{s.t.} \quad \text{PPF}$$

- The PPF summarizes all that can be produced using labor efficiently.
- Robinson Crusoe chooses the quantities of goods 1 and 2 to maximize his utility subject to the PPF.

Graphically



Comments

- At the optimal choice of (x_1, x_2):

$$\text{MRS} = \text{MRT}$$

- No markets in this economy. Just choosing the best thing to do given the total endowment $\bar{L}$.
- No prices in this problem.
- $\text{MRT} = \frac{\text{MP}_2}{\text{MP}_1}$ as the ratio of marginal products.

Market Economy

Markets

- We introduce markets:
  - Market of good 1 (fish). Its price is p_1.
  - Marge of good 2 (coconuts). Its price is p_2.
  - Labor market. Its price is $w = 1$.
- Goal: Illustrate how a market economy will deliver the same allocation.
- Insights on how people following only their own interest generate the efficient allocation.
- Focusing on 2 (out of the 3) markets is enough.

Robinson: Producer

$$\max_{l_1} \quad p_1 \cdot f_1(l_1) - l_1$$

- Good 1. Profits: $\pi_1(p_1)$. Supply of good 1: $y_1(p_1)$.

$$\max_{l_2} \quad p_2 \cdot f_2(l_2) - l_2$$

- Good 2. Profits: $\pi_2(p_2)$. Supply of good 2: $y_2(p_2)$.

Robinson: Consumer

$$\max_{x_1,x_2} \quad u(x_1,x_2) \quad \text{s.t.} \quad p_1x_1 + p_2x_2 = \bar{L} + \pi_1(p_1) + \pi_2(p_2)$$

- Robinson has labor income $\bar{L}$.
- And he is the owner of the firms. Profits are redistributed to him.
- Demands for goods 1 and 2: $x_1(p_1,p_2)$ and $x_2(p_1,p_2)$.

Equilibrium

- Consider the markets of fish and coconuts. Set supply equal to demand and determine prices p_1^* and p_2^*.
- Because Robinson the consumer is maximizing utility:

$$\text{MRS} = \frac{p_1^*}{p_2^*}$$

- Because Robinson the producer is maximizing profits:

$$p_1^* \cdot \text{MP}_1 = 1 \quad \text{and} \quad p_2^* \cdot \text{MP}_2 = 1 \implies \text{MRT} = \frac{p_1^*}{p_2^*}$$

Comments

- Equilibrium are prices $(p_1^*, p_2^*, w^* = 1)$, quantities produced and sold (x_1^*, x_2^*), and amount of labor in each industry (l_1^*, l_2^*) such that $l_1^* + l_2^* = \bar{L}$.
- Prices aggregate dispersed information. In equilibrium they are such that MRS is equal to MRT.
- (First Welfare Theorem) In equilibrium there are no further market opportunities that would make someone better off without making other individual worse off.

Extensions

Disutility of Labor

- Robinson chooses how much of the total endowment of time $\bar{L}$ to spend working l or enjoying leisure h. Of course $l + h = \bar{L}$.
- $u(x_1, x_2, h)$, where leisure h is a third good.
- Three markets:
  1. Fish x_1, price p_1.
  2. Coconuts x_2, price p_2.
  3. Labor (or leisure) market l (or h), price w.
- Before, the labor market was straightforward.
- Now, the labor market requires some work.

More Factors

- In addition to a total endowment of time to work $\bar{L}$, Robinson owns a total endowment of capital to employ in production $\bar{K}$.
- Production functions $y_1 = f_1(k, l)$ and $y_2 = f_1(k, l)$ such that $k_1 + k_2 = \bar{K}$ and $l_1 + l_2 = \bar{L}$.
- Four markets:
  1. Fish x_1, price p_1.
  2. Coconuts x_2, price p_2.
  3. Labor l, price w.
  4. Capital k, price r
- Robinson the consumer owns capital and labor, so the returns r and w go to him.

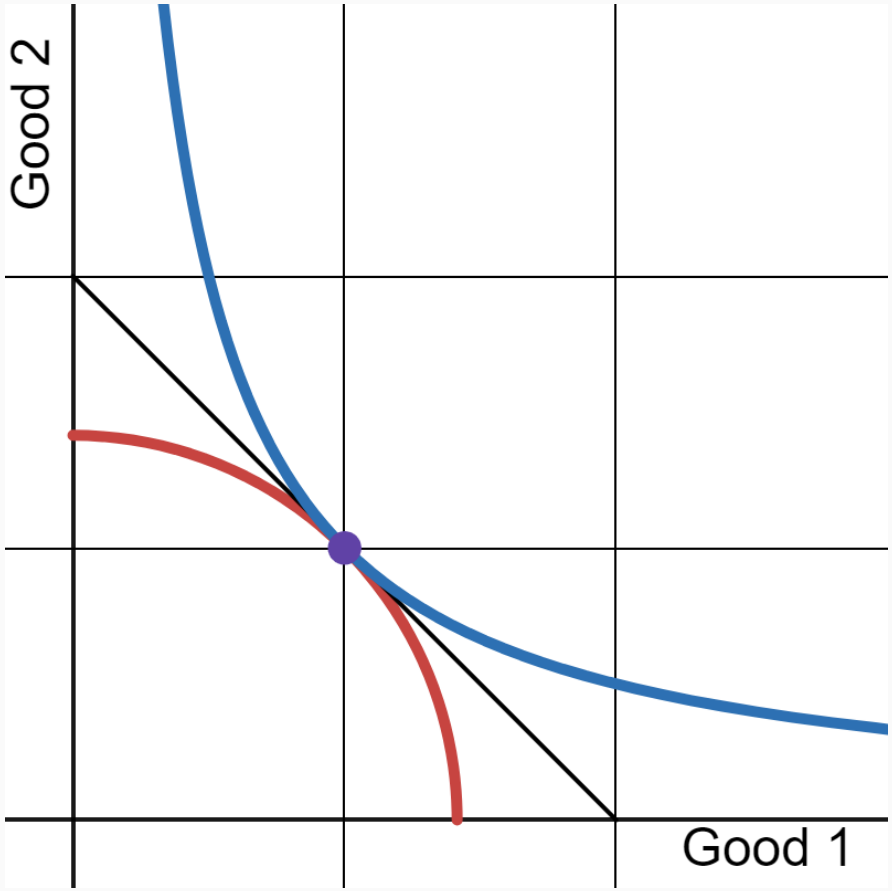
More People

- Now there are 2 or more people.
- Each person maximizes their utility and this produces the individual demand for each good (and individual labor supply).
- Same three markets:
  1. Fish x_1, price p_1.
  2. Coconuts x_2, price p_2.
  3. Labor l, price w.
- We need to consider the aggregate demand to pin down the equilibrium prices.
- More firms? Same idea.

International Trade

- We can think of Robinson and his island representing the production in a closed economy.
- What if there is another island that offers engaging into trade for some given prices?
- Robinson specializes in one of the goods and takes advantage of trade to increase his utility.
- We take the international prices as given.
- What pins down international prices? Take an international economics class.

International Trade: Closed Economy



International Trade: Open Economy