General Equilibrium: Exchange

Javier Tasso

June 14, 2024

University of Pennsylvania

Endowments & Preferences

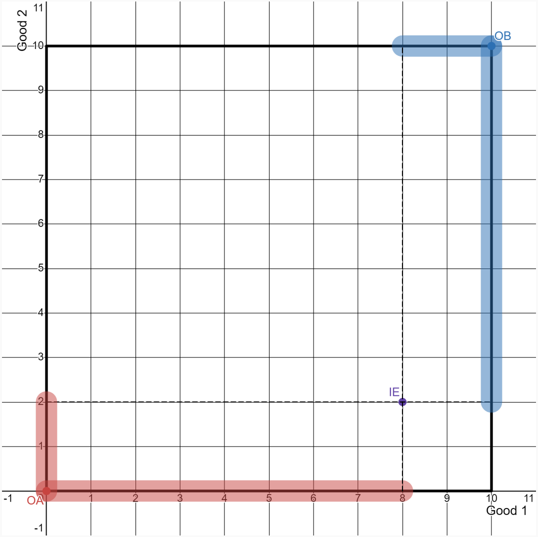
Endowments

- Two goods.
- No production in this model.
- ω_1 represents the total endowment of good 1 there is in the economy.
- ω_2 represents the total endowment of good 2 there is in the economy.

Endowments II

- Two consumers A and B.
- We know their individual endowments:
    - (ω_1^A, ω_2^A) represents how many units of goods 1 and 2 are owned by A.
    - (ω_1^B, ω_2^B) represents how many units of goods 1 and 2 are owned by B.
    - Of course $\omega_1^A + \omega_1^B = \omega_1$ and $\omega_2^A + \omega_2^B = \omega_2$. They add up to the total endowment.
- We can plot the endowments in the Edgeworth box.

Endowments III



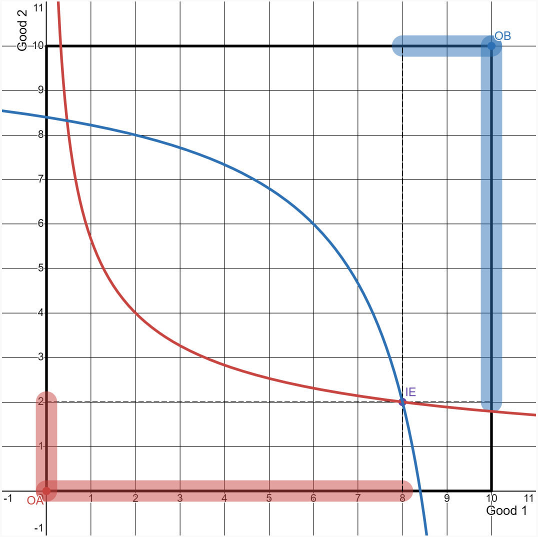
Allocations

- An allocation is a pair of vectors $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$ that specifies consumption of goods 1 and 2 for agents A and B.
- An allocation is feasible if $x_1^A + x_1^B \leq \omega_1$ and $x_2^A + x_2^B \leq \omega_2$. In other words, if it's inside the Edgeworth box.

Preferences

- Consumer A has preferences represented by $u^A(x_1, x_2)$.
- Consumer B has preferences represented by $u^B(x_1, x_2)$.
- We can plot their indifference curves on the Edgeworth box.

Preferences II



Equilibrium

Equilibrium Definition

An allocation $x^{A*} = (x_1^{A*}, x_2^{A*})$, $x^{B*} = (x_1^{B*}, x_2^{B*})$ and prices (p_1^*, p_2^*) constitute a competitive equilibrium if:

- x^{A*} solves the utility maximization problem of consumer A taking prices (p_1^*, p_2^*) as given.
- x^{B*} solves the utility maximization problem of consumer B taking prices (p_1^*, p_2^*) as given.
- The allocation is feasible.

UMP

$$\max_{x_1,x_2} \quad u^A(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = p_1 \omega_1^A + p_2 \omega_2^A$$

- His ordinary demands are:

$$x_1^A(p_1, p_2) \quad \text{and} \quad x_2^A(p_1, p_2)$$

- Similarly for consumer B:

$$x_1^B(p_1, p_2) \quad \text{and} \quad x_2^B(p_1, p_2)$$

Excess Demand

- Define the excess demand of good 1:

$$z_1(p_1, p_2) = x_1^A(p_1, p_2) + x_1^B(p_1, p_2) - \omega_1^A - \omega_1^B$$

- And for good 2:

$$z_2(p_1, p_2) = x_2^A(p_1, p_2) + x_2^B(p_1, p_2) - \omega_2^A - \omega_2^B$$

Excess Demand II

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

- Why?
- Walras' Law.
- Always holds. Even if (p_1, p_2) are not the equilibrium prices.
- The sum of (the value of) excess demand is always zero.

Walras' Law Corollary

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

- If one of the markets is in equilibrium $(z_i(p_1, p_2) = 0)$, the other market is also in equilibrium.
- It suffices to analize only one market (Or $n - 1$ if there are n markets).

Relative Prices

- $z_i(p_1, p_2)$ are homogeneous of degree 0. Why?
- They are also continuous. Why?
- This model will determine only relative prices p_1/p_2.
- We can normalize prices such that $p_1 + p_2 = 1$. We stick with this in the theory.
- Or we can set $p_2 = 1$ (or any other value). We do this in exercises.

Characterization

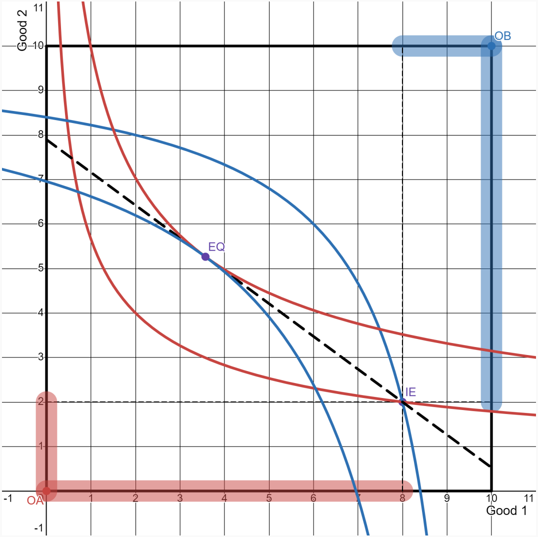
- At the equilibrium allocation both consumers are maximizing their utility subject to their budget constraint.
- Also they face the same prices.

$$\mathrm{MRS}_A = \frac{p_1^*}{p_2^*} \quad \text{and} \quad \mathrm{MRS}_B = \frac{p_1^*}{p_2^*}$$

- This implies:

$$\mathrm{MRS}_A = \mathrm{MRS}_B$$

Illustration



Existence

Equilibrium Prices

$$z(p) = (z_1(p_1, p_2), z_2(p_1, p_2))'$$

- Shortcut notation: stack the two excess demand functions.
- Set $p = (p_1, p_2)$ with $p_1 + p_2 = 1$.
- The equilibrium price is such that $z(p^*) = \vec{0}$.
- Let's prove such a price exists.

Equilibrium Existence

- Let $k > 0$ be a small constant.

$$p' = p + kz(p)$$

- If there's excess demand for good 1 $(z_1(p) > 0)$ its price goes up.
- If there's excess supply for good 1 $(z_1(p) < 0)$ its price goes down.
- Similar for good 2.
- This is Walras' tâtonnement (French for "trial and error") process.

Equilibrium Existence II

- Brouwer's Fixed Point Theorem.
- Any continuous function must cross the 45 degree line somewhere in the unit square.
- See graph.
- We find:

$$p^* = p^* + kz(p^*) \implies z(p^*) = 0$$

- So $p^* = (p_1^*, p_2^*)$ are the equilibrium prices.

Efficiency

Efficiency

- An allocation $x = (x^A, x^B)$ is Pareto efficient if there is no other allocation $\hat{x} = (\hat{x}^A, \hat{x}^B)$ such that: $u_A(\hat{x}^A) \geq u_A(x^A)$ and $u_B(\hat{x}^B) \geq u_B(x^B)$ and some inequality is strict.
- In words, if there is no other allocation where both agents are at least as well off, but someone is strictly better off.
- If such allocation exists, we often call it a Pareto improvement.

Efficiency II

- Pareto improvements are desirable: they mean we can make someone better off, without harming no one else.
- Is the IE in the previous Edgeworth box efficient?
- An allocation that gives everything of both goods to consumer A and nothing to B is Pareto efficient. Why?
- Efficiency vs equality. Pareto efficiency doesn't say anything about equality.
- Efficiency just says nothing goes to waste.

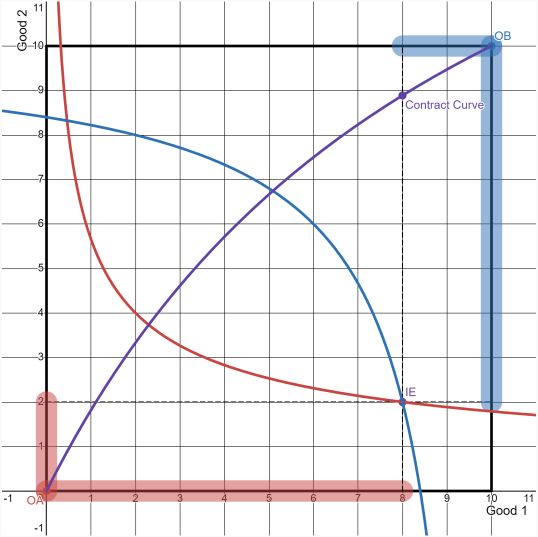
Efficiency III

$$\max_{x_1^A, x_2^A, x_1^B, x_2^B} \quad \alpha u^A(x_1^A, x_2^A) + (1-\alpha) u^B(x_1^B, x_2^B)$$

$$\text{s.t.} \quad \begin{cases} x_1^A + x_1^B = \omega_1 \\ x_2^A + x_2^B = \omega_2 \end{cases}$$

- $\alpha \in [0, 1]$.
- If we solve this problem, we find all the Pareto efficient allocations.
- If we plot the answer for different α, we get the Pareto set or contract curve.

Contract Curve or Pareto Set



Efficiency

- Another characterization of the Pareto set. Whenever possible.

$$\mathrm{MRS}^A = \mathrm{MRS}^B$$

- Why is this the case?
    - Suppose $\mathrm{MRS}^A < \mathrm{MRS}^B$.
    - If I take one unit of good 1 away from A, I can compensate him with little good 2.
    - If I take one unit of good 2 away from B, I can compensate him with little good 1.
    - Then we can improve both consumers by giving more good 2 to A and more good 1 to B.

Welfare Theorems

First Theorem of Welfare Economics

- First Theorem of Welfare Economics.
- Statement: *Every competitive equilibrium is Pareto efficient.*

Second Theorem of Welfare Economics

- Statement: *For any Pareto optimal allocation of resources there exists a set of initial endowments and a related price vector such that this allocation is also a competitive equilibrium.*
- Any Pareto optimal allocation can be a competitive equilibrium, provided that the initial endowments are adjusted accordingly.