ECON 2100 - Intermediate Microeconomics Problem Set #2
Javier Tasso.

Please submit a single PDF file with your name on it. Cooperation between students is allowed and encouraged, but each student must submit its own solution. Please write down the name of the classmate/s you cooperated with, if any.

Problem 1. Exchange economy with quasi-linear preferences. Consider individuals A and B with preferences and endowments described below. During this exercise you can assume all solutions are interior. There are in total 12 units of good 1 and 10 units of good 2. Out of the aggregate supply of 12 units of good 1, consumer A owns ω and consumer B owns the rest. The price vector is $(p_1, p_2) = (1, p)$, so we normalize the price of good 1.

$$u_A(x_1^A, x_2^A) = x_1^A + \ln(x_2^A) \quad \text{with} \quad \omega_A = (\omega, 0)$$
$$u_B(x_1^B, x_2^B) = x_1^B + \ln(x_2^B) \quad \text{with} \quad \omega_B = (12 - \omega, 10)$$

1. For each consumer solve the utility maximization problem. Find individual demands $x_1^A(p, \omega)$, $x_2^A(p, \omega)$, $x_1^B(p, \omega)$, and $x_2^B(p, \omega)$ as a function of the relative price p and the parameter ω.
2. Focus on good 2. Find the aggregate demand. Find the equilibrium level of the relative price p. Does it depend on ω?
3. Find the contract curve or Pareto set.
4. Suppose $\omega = 2$, find the equilibrium quantities and utility levels for each consumer.
5. Choose a value for ω such that both consumers reach the same utility level. Find the equilibrium quantities.
6. Plot all your previous answers in an Edgeworth box. This includes: the initial endowment, the equilibrium allocation with the final indifference curves, the budget line, and the contract curve. Your answer should include one Edgeworth box for 4 and another one for 5.
7. Economists often say that in this context transfering units of good 1 between consumers is equivalent to transferring utility between consumer. What feature of the utility function delivers this result? Explain.

Problem 2. Production economy and trade. Robinson Crusoe lives alone in an island. There are two goods 1 and 2 with production functions $x_1(l_1) = 5\sqrt{l_1}$ and $x_2(l_2) = l_2$, where l_1 and l_2 represent the hours of work used in production. Robinson's utility function is given by $u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$ (note he does not get utility out of leisure, so he'll work as much as possible).

1. There are in total 20 hours a day he can work. Find the production possibility frontier and the marginal rate of transformation (MRT).

2. Calculate Robinson's marginal rate of substitution (MRS). Obtain optimal consumption and production. What's the maximum utility level he attains?

3. Plot the production possibilities frontier together with the indifference curve at the maximum utility attained. What is the rate of change between goods 1 and 2?

4. Find the equilibrium prices of the three markets ($p_1^*, p_2^*, w^* = 5$) and the amount of labor used in the production of each good. The normalization is $w^* = 5$.

5. Suppose Robinson is allowed to trade with another island. They can trade goods at a rate 8/5, so 5 units of good 1 per 8 units of good 2. Find the optimal production at this rate.

6. Find the optimal consumption at this rate. How many goods do they exchange? Hint: set $p_1 = 8$ and $p_2 = 5$ and find Robinson's budget constraint using the production from part 5. What's the maximum utility level?

7. Plot your answers of parts 5 and 6.

Problem 3. Intertemporal Choice Revisited. A consumer lives for three periods: young age (denoted as $t = 0$), adulthood (denoted as $t = 1$), and old age (denoted as $t = 2$).

- In her young age she studies and she does not make any income.
- In her adulthood she works and gets a total income of m.
- In her old age she stops working and does not make any income.

The problem is to choose her lifetime consumption. Let $0 < \beta < 1$ be the discount factor and $r \geq 0$ the real interest rate.

$$\max_{c_0, c_1, c_2} \quad \ln(c_0) + \beta \ln(c_1) + \beta^2 \ln(c_2) \quad \text{s.t.} \quad c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} = \frac{m}{1+r} \tag{1}$$

1. Solve the utility maximization problem. Your answer should specify consumption functions $c_t(\beta, r, m)$ for each moment in life $t = 0, 1, 2$. How do each of them depend on r? Explain.

2. For the rest of the exercise assume $m = 2100$, $r = 0.25$, and $\beta = 0.5$. Complete the following table. What is the maximum utility she gets in this situation? For the forth and fifth column, you may use positive numbers for savings and negative numbers for debt.

3. Now there is a borrowing constraint. The consumer can only borrow up to 480 in $t = 0$. Assume then that $c_0 = 480$ and her debt at the end of $t = 0$ is 480. What's her debt at the beginning of $t = 1$? We call it D.

	Income	Consumption	Initial Debt/Savings	Final Debt/Savings
$t = 0$	0		0	
$t = 1$	2100			
$t = 2$	0			0

4. Solve the following reduced problem at $t = 1$. You should plug in D you found previously. $2100-D$ is meant to represent that she pays her debt and uses the remaining income to consume and save. Find the optimal consumption levels.

$$\max_{c_1,c_2} \quad \ln(c_1) + \beta \ln(c_2) \quad \text{s.t.} \quad c_1 + \frac{c_2}{1+0.25} = 2100 - D$$

5. Use your answers of part 4 to complete the following table. Calculate the utility the consumer gets in this constrained situation. For the utility calculation, you should use the utility of problem (1), and the consumption of the table.

	Income	Consumption	Initial Debt/Savings	Final Debt/Savings
$t = 0$	0	480	0	-480
$t = 1$	2100			
$t = 2$	0			0

Problem 4. Insurance. A consumer has the following Bernoulli utility function defined over her wealth, in this case, a house. Her house is worth $w = 36$, but with probability $1/2$ a flooding happens completely destroying it and its value falls to $w = 0$.

$$u(w) = w^{1/2}$$

There is the possibility of insurance: If the flooding does not happen, her wealth is $36 - rx$. If the flooding happens, her wealth is $0 - rx + x$. Where x is the amount of wealth she chooses to insure, and r the price she pays. This means she pays rx to the insurance company no matter what, but if the flooding happens, she receives x back.

1. Find the Arrow-Pratt coefficient of relative risk aversion. Is this agent risk averse?

2. Calculate her expected utility without insurance.

3. Find an expression for the expected utility with insurance. Your expression will depend on r and x.

4. Find the amount of insurance x that maximizes the expected utility of part 3. Your answer will depend on r. Call it $x(r)$. This is the demand for insurance.

(a) Find the range of r for which $x(r)$ is non-negative. From now on, you can assume r is always within that range.

5. An insurance company. Its expected profits are given below.

$$\mathbb{E}[\pi] = rx(r) - \frac{1}{2}x(r) = \left(r - \frac{1}{2}\right) \cdot x(r)$$

Calculate:

(a) The range of values for which the expected profits are non-negative.

(b) The price r^* that makes expected profits equal to zero. The insurance company is competitive in this situation. How much insurance does the consumer buy?

(c) The price r^M that maximizes expected profits. The insurance company is a monopolist here. How much insurance does the consumer buy?