ECON 2100 - Intermediate Microeconomics Problem Set #1

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Please submit a single PDF file with your name on it. Cooperation between students is allowed and encouraged, but each student must submit its own solution. Please write down the name of the classmate/s you cooperated with, if any.

Problem 1. UMP and EMP. Consider the following four utility functions. The consumption set is $X = \mathbb{R}^2_+$. α and β are positive real numbers.

- Cobb Douglas: $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ for $0 < \alpha < 1$.
- Perfect Complements: $u(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$.
- Perfect Substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$.
- Quasi-linear: $u(x_1, x_2) = \ln(x_1) + x_2$.

For each of them:

1. Solve the utility maximization problem: Find $x_1(p_1, p_2, m)$, $x_2(p_1, p_2, m)$, and $v(p_1, p_2, m)$ that solve $\max_{x_1,x_2} u(x_1, x_2)$ s.t. $p_1x_1 + p_2x_2 = m$.
2. Solve the expenditure minimization problem: Find $h_1(p_1, p_2, u)$, $h_2(p_1, p_2, u)$, and $e(p_1, p_2, u)$ that solve $\min_{x_1,x_2} p_1x_1 + p_2x_2$ s.t. $u(x_1, x_2) = u$.

Problem 2. Dead-weight loss of a tax. Consider the following indirect utility function.

$$v(p_1, p_2, m) = \frac{4m^3}{27p_1^2 p_2}$$

1. Find the Marshallian demands $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.
2. Find the expenditure function $e(p_1, p_2, u)$.

Initially $(p_1, p_2, m) = (1, 1, 12)$, but the government set a tax of 0.6 per unit of good 1 such that its price rises to $p_1' = 1.6$.

3. Calculate the consumption levels and the utility achieved before and after the tax.
4. Show that the tax raises $3 of revenue.
5. The commodity tax we are discussing implies $(p_1, p_2, m) = (1.6, 1, 12)$ while a lump sum tax of $3 implies $(p_1, p_2, m) = (1, 1, 9)$. Which tax does the consumer prefer?

6. Calculate $e(1, 1, u')$ where u' is the utility level after the tax from question 3. We say the commodity tax is equivalent to a lump sum tax of $12 - e(1, 1, u')$.

The difference between your answers in parts 6. and 4. is the dead-weight loss of the tax. This is welfare lost by the consumer that was not captured in the tax revenue.

Problem 3. Intertemporal choice. A consumer has an income of \$240 and must decide how much to save for retirement. Her preferences and budget constraints are given below. In what follows we assume there are two periods (0 for today and 1 for the retirement period) $0 \leq \beta \leq 1$ is the discount factor and r the real interest rate. c_0 is the consumption of a representative good today and c_1 is the consumption in the retirement period. The price level is normalized.

$$u(c_0, c_1) = \ln(c_0) + \beta \ln(c_1)$$
$$c_0 + \frac{c_1}{1+r} = 240$$

1. Discuss intuitively how the saving decisions of an agent with β close to 0 will differ from the saving decisions of an agent with a β close to 1.
2. The budget constraint is in present value: the left hand side represents the present value of consumption and the right hand side the present value of the income. Discuss intuitively what is the role of the interest rate r in the budget constraint.
3. Find $c_0(\beta, r)$ and $c_1(\beta, r)$ that solve the utility maximization problem. Make comparative statics with respect to β and r.
4. For $\beta = 0.5$ and $r = 0.25$, calculate the consumption in each period and the amount of savings.
5. This agent likes to smooth consumption over her lifetime instead of consuming everything in the first period. What assumption on preferences is responsible for this result?

Problem 4. Labor supply. An individual solves the following utility maximization problem.

$$\max_{c,h,l} \quad ch \quad \text{s.t.}$$
$$c = wl + n$$
$$h + l = 1$$

Where c is consumption, h is leisure, l is labor, w is the real wage, and n is the non-labor income. The length of the day is normalized to 1. The problem of this consumer is to choose what fraction of the day to work and how much to consume to maximize her utility $u(c, h) = ch$.

1. Solve the problem. Your answer should include consumption demand $c(w, n)$, leisure demand $h(w, n)$, and labor supply $l(w, n)$. How does labor supply respond to changes to w and n? Hint: you can substitute the constraints and eliminate variables.
2. Let $(w, n) = (10, 0)$. In a graph with h in the horizontal axis and c in the vertical axis plot the budget constraint and the highest indifference curve attained.
3. Let $(w, n) = (20, 0)$. Repeat the previous question.
4. Let $(w, n) = (10, 4)$. Repeat the previous question. Careful: leisure h cannot exceed 1.

For the rest of the exercise assume $n = 0$.

5. Find the compensated labor supply $l^C(w, u)$ that solves the expenditure minimization problem below. Hint: you can eliminate consumption by substituting the constraint.

$$\min_{c,l} \quad c - wl \quad \text{s.t.} \quad c(1 - l) = u$$

6. Suppose the wage goes from $w = 10$ to $w = 20$. Decompose the total change in labor supply into income and substitution effect.