Microeconomics
Homework 7: General Equilibrium - Production

Javier Tasso

1. Consider an one agent economy. His preferences over coconuts x and leisure h are $u(x,h) = xh$. The initial endowment is $(0, 24)$, so 0 coconuts and 24 hours of leisure. Coconuts are produced according to $f(l) = \sqrt{l}$, where l is hours of work.

   (a) Find the efficient allocation by solving the following optimization problem.

   $$\max_{x,h} \quad xh \quad \text{s.t.} \quad x = \sqrt{l} \quad \text{and} \quad h + l = 24$$

   Now we move to a market economy. Normalize $w = 1$ and let p be the (relative) price of coconuts.

   (b) Solve the profit maximization problem. Find the demand for labor $l(p)$, the supply of coconuts $y(p)$, and the profit function $\pi(p)$.

   (c) Solve the utility maximization problem. Because this agent is the owner of the firm, he gets all the profits on top of labor income. Find the demand of coconuts $x(p)$ and supply of labor $l^s(p)$.

   (d) Find the equilibrium price and allocation.

2. In this single agent economy there are two goods and labor. Good 1 is produced according to $f_1(l_1) = 5\sqrt{l_1}$ and good 2 according to $f_2(l_2) = l_2$. Preferences are given by $u(x_1, x_2) = x_1 x_2^2$. There are 20 hours in a day and since this person does not care about leisure, we know $l_1 + l_2 = 20$.

   (a) Find the production possibility frontier and the marginal rate of transformation MRT.

   (b) Find the MRS of the consumer and use it to find optimal consumption and production. What's the maximum utility attained?

   (c) Plot the indifference curve as well as the production possibility frontier.

3. Two individuals each have 10 hours of labor to devote to producing x_1 or x_2. Utility and production functions are given below.

   $$u_A(x_1, x_2) = x_1^{0.3} x_2^{0.7}$$
   $$u_B(x_1, x_2) = x_1^{0.5} x_2^{0.5}$$
   $$y_1(l) = 2l$$
   $$y_2(l) = 3l$$

   (a) What must the price ratio p_1/p_2 be? Hint: find the MRT between goods.

   (b) Normalize $w = 1$ and given the price ratio, how much x_1 and x_2 will A and B demand?

(c) How should labor be allocated between goods 1 and 2 to satisfy the demand calculated in (b)?

4. Robinson Crusoe produces and consumes fish x_1 and coconuts x_2. There 200 hours a month available to work. Production and utility functions are given below.

$$
\begin{aligned}
f_1(l) &= \sqrt{l} \\
f_2(l) &= \sqrt{l} \\
l_1 + l_2 &= 200 \\
u(x_1, x_2) &= x_1^{0.5} x_2^{0.5}
\end{aligned}
$$

(a) How does Robinson choose to allocate labor? Find the optimal levels of x_1 and x_2. Find the utility level. Find the MRT.

(b) Suppose now that trade is opened and Robinson can trade fish and coconuts at a price ratio of $p_1/p_2 = 2$. If Robinson continues to produce the same quantities you found in (a), what will he choose to consume? What's his new level of utility?

(c) Now assume Robinson adjusts the production in order to take advantage of trade. Find the production, consumption, and utility.

(d) Plot your answers.

5. (Varian 18.2) Consider an economy with two firms and two consumers. Firm 1 is entirely owned by consumer A.

$$y_1 = f_1(l) = 2l$$

Firm 2 is entirely owned by consumer B.

$$y_2 = f_2(l) = 3l$$

Each consumer owns 10 units of labor. Their utilities are:

$$u_A(x_1, x_2) = x_1^{0.4} x_2^{0.6} \quad \text{and} \quad u_B(x_1, x_2) = x_1^{0.5} x_2^{0.5}$$

(a) Normalize $w = 6$. Find the market clearing prices of goods 1 and 2.

(b) Find the optimal consumption for each consumer.

(c) How much labor does each firm use?

Answers

1. (a) $l^* = 8$, $h^* = 16$, and $x^* = 2\sqrt{2}$.
   (b) $l(p) = \frac{p^2}{4}$, $y(p) = \frac{p}{2}$, and $\pi(p) = \frac{p^2}{4}$
   (c) $l^s(p) = 12 - \frac{p^2}{8}$ and $x(p) = \frac{12}{p} + \frac{p}{8}$
   (d) $p^* = 4\sqrt{2}$, $x^* = 2\sqrt{2}$, and $l^* = 8$.

2. (a) PPF: $x_2 = 20 - \frac{x_1^2}{25}$. The MRT is $\frac{2x_1}{25}$
   (b) The MRS is $\frac{x_2}{2x_1}$, the optimal consumption is $x_1^* = 10$ and $x_2^* = 16$.
   (c) See figure.

3. (a) $p_1/p_2 = 3/2$.
   (b) Consumer A buys $(6, 21)$ and consumer B buys $(10, 15)$. In competitive markets real wage w/p_1 must be equal to marginal product of labor. Set $w = 1$ and solve for p_1. The same is true for p_2.
   (c) We need a total of 16 units of good 1 and 36 units of good 2. So we need to employ 8 hours of work on producing x_1 and 12 hours of work on producing x_2.

4. (a) $l_1^* = l_2^* = 100$, $x_1^* = x_2^* = 10$, $u^* = 10$, and MRT $= 1$. The expression for hte MRS is x_2/x_1. The expression for the MRT is x_1/x_2. The expression for the PPF is $x_1^2 + x_2^2 = 200$. Setting MRS $=$ MRT and using the PPF delivers the optimal consumption.
   (b) The budget constraint is $2x_1 + x_2 = 30$. Setting MRS equal to the price ratio gives you $x_2 = 2x_1$. Then the optimal consumption is $x_2 = 15$ and $x_1 = 7.5$. Here $u \simeq 10.606$.
   (c) From the PPF using $x_1 = 2x_2$ we get production: $y_1 = 4\sqrt{10}$ and $y_2 = 2\sqrt{10}$. The new budget constraint is $2x_1 + x_2 = 10\sqrt{10}$ which gives optimal consumption (using $x_2 = 2x_1$) $x_1 = \frac{5}{2}\sqrt{10}$ and $x_2 = 5\sqrt{10}$. Utility is $u \simeq 11.18$.
   (d) See figure.

5. (a) $(p_1, p_2, w) = (3, 2, 6)$. Set marginal product of labor equal to the real wage.
   (b) For consumer A: $(8, 18)$ and for consumer B: $(10, 15)$.
   (c) Firm 1 uses $l = 9$ and firm 2 uses $l = 11$.

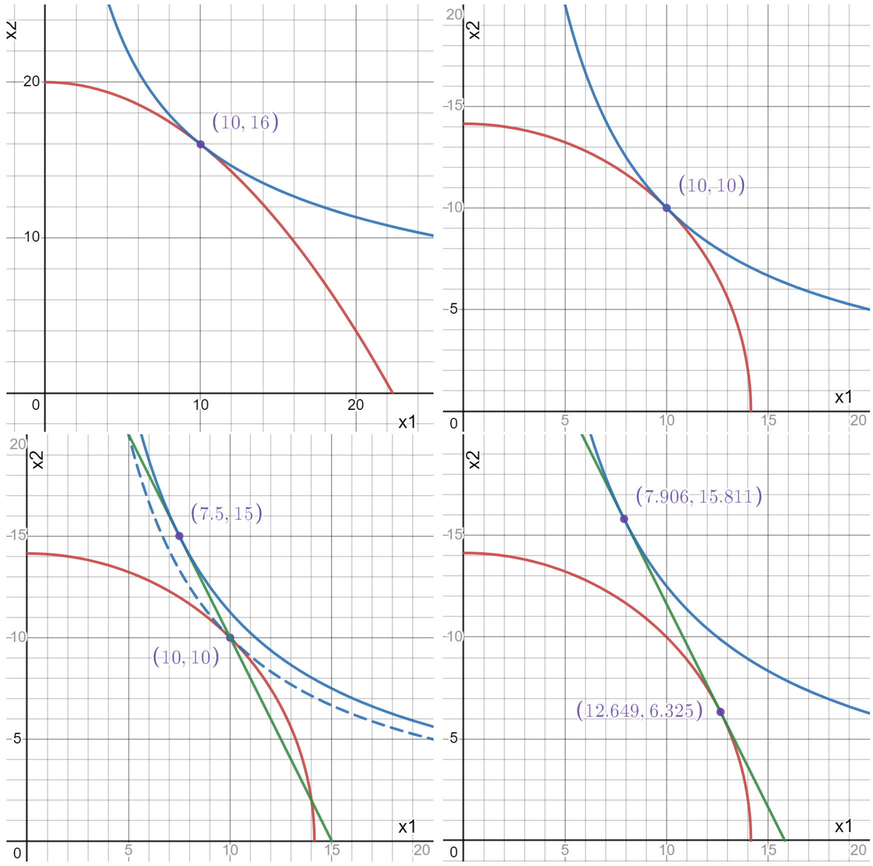

Figure 1: Ex 2 (top left) and Ex 4 (all the others)