Microeconomics
Homework 4: Welfare Analysis

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1. Consider the utility function $u(x_1, x_2) = x_1 x_2$. Initially $(p_1, p_2, m) = (1, 1, 12)$. Suddenly the price of good 1 changes to $p_1' = 3$.
   (a) Calculate the compensate and equivalent variations using the expenditure function.
   (b) Show your answers in a graph with x_1 and x_2 in the axis.
   (c) Show your answers in a graph with x_1 in the horizontal axis and p_1 in the vertical axis.
   (d) From the last graph: What's the change in the consumer surplus?

2. Consider the utility function $u(x_1, x_2) = \min\{x_1, x_2\}$. Initially $(p_1, p_2, m) = (1, 1, 12)$. Suddenly the price of good 1 changes to $p_1' = 3$.
   (a) Calculate the compensate and equivalent variations using the expenditure function.
   (b) Show your answers in a graph with x_1 and x_2 in the axis.
   (c) Show your answers in a graph with x_1 in the horizontal axis and p_1 in the vertical axis.
   (d) From the last graph: What's the change in the consumer surplus?

3. Consider the utility function $u(x_1, x_2) = x_1 + x_2$. Initially $(p_1, p_2, m) = (1, 3, 12)$. Suddenly the price of good 1 doubles to $p_1' = 2$. Calculate the compensating and equivalent variation.

4. Repeat the previous exercise, but instead of p_1 doubling, it is 4 times as high. So $p_1' = 4$.

5. Consider the utility function $u(x_1, x_2) = \ln(x_1) + x_2$. Initially $(p_1, p_2, m) = (1, 1, 4)$, but suddenly $p_1' = 2$.
   (a) Calculate the compensate and equivalent variations using the expenditure function.
   (b) Show your answers in a graph with x_1 and x_2 in the axis.
   (c) Show your answers in a graph with x_1 in the horizontal axis and p_1 in the vertical axis.
   (d) From the last graph: What's the change in the consumer surplus?

Answers

1. The initial and final utility levels are respectively 36 and 12.

   (a) CV $\simeq -8.78$ and EV $\simeq -5.07$

   (b) See figure. Because $p_2 = 1$ we read the EV and CV directly from the x_2 axis.

   (c) See figure for the EV. The CV is the (negative) area between the blue hicksian and the y-axis.

   (d) We want to calculate the area between the marshallian demand (red) and the axis. The change in the consumer surplus is then $\int_3^1 \frac{6}{p} dp = -6.59$. Note that lies between the other two values.

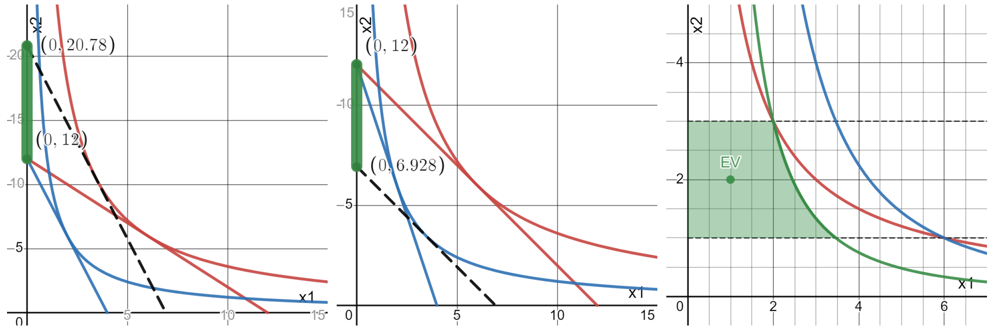

Figure 1: Exercise 1

2. The initial utility levels are respectively 6 and 3.

   (a) CV $= -12$ and EV $= -6$.

   (b) See figure. Because $p_2 = 1$ we read the amounts directly from the x_2-axis.

   (c) See figure for the EV. The CV is the (negative) area between the blue hicksian and the y-axis.

   (d) Calculate the area between the marshallian demand (in red) and the y-axis. $\int_3^1 \frac{12}{p+1} dp = -8.32$. Note that this lies between the other two values.

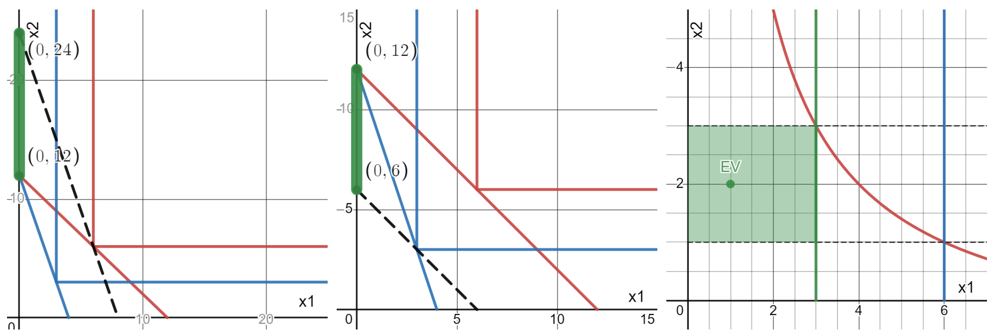

Figure 2: Exercise 2

3. CV $= -12$ and EV $= -6$.

4. CV $= -24$ and EV $= -8$.

5. The initial and final utility levels are 3 and 2.31.

   (a) CV $= -0.6931$ and EV $= -0.6931$.

   (b) See figure. Because $p_2 = 1$ we read the amounts derectly from the x_2-axis.

(c) See figure. Only one demand is needed because in this case the hicksian and the marshallian demand are the same. Additionally, the hicksian demand does not depend on the utility level.

(d) No need to calculate the change in the consumer surplus. It will be -0.6931

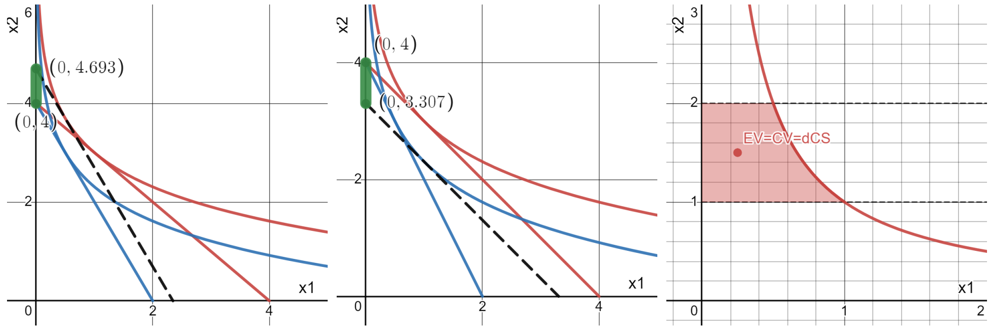

Figure 3: Exercise 5