# Microeconomics
# Homework 3: Expenditure Minimization Problem & Slutsky Equation

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1. Given the following utility functions:

   - Cobb-Douglas: $u(x_1, x_2) = x_1 x_2^3$
   - Perfect Substitutes: $u(x_1, x_2) = x_1 + 3x_2$
   - Perfect Complements: $u(x_1, x_2) = \min\{x_1, 3x_2\}$
   - Quasi-Linear: $u(x_1, x_2) = \ln(x_1) + x_2$

   (a) Solve the expenditure minimization problem. Your answer should include $h_1(p_1, p_2, u)$, $h_2(p_1, p_2, u)$, and $e(p_1, p_2, u)$.

   (b) Verify Shephard lemma.

   (c) Verify that given prices $p_1$ and $p_2$, $e(p_1, p_2, \cdot)$ is the inverse of $v(p_1, p_2, \cdot)$.

2. Given the following utility functions:

   - Cobb-Douglas: $u(x_1, x_2) = x_1 x_2$
   - Perfect Substitutes: $u(x_1, x_2) = x_1 + x_2$
   - Perfect Complements: $u(x_1, x_2) = \min\{x_1, x_2\}$

   (a) Solve the expenditure minimization problem. Your answer should include $h_1(p_1, p_2, u)$, $h_2(p_1, p_2, u)$, and $e(p_1, p_2, u)$.

   (b) Verify Shephard lemma.

   (c) Verify that given prices $p_1$ and $p_2$, $e(p_1, p_2, \cdot)$ is the inverse of $v(p_1, p_2, \cdot)$.

3. Consider the utility function $u(x_1, x_2) = x_1 x_2$. Initially $(p_1, p_2, m) = (1, 1, 12)$. Suddenly the price of good 1 changes to $p_1' = 3$. Decompose total change in the demand of good 1 into income and substitution effect.

4. Same as the previous exercise, but decompose the total change in the demand of good 2.

5. Same setting as the previous two exercises. In the same graph, plot the hicksian (at the original utility level) and marshallian demands for good 1.

6. Consider the utility function $u(x_1, x_2) = \min\{x_1, x_2\}$. Initially $(p_1, p_2, m) = (1, 1, 12)$. Suddenly the price of good 1 changes to $p_1' = 3$. Decompose the change in the demand of good 1 into income and substitution effect.

7. Same setting as the previous exercise In the same graph, plot the hicksian and marshallian demands for good 1.

8. Consider the utility function $u(x_1, x_2) = x_1 + x_2$. Initially $(p_1, p_2, m) = (1, 3, 12)$. Suddenly the price of good 1 doubles to $p_1' = 2$. Decompose the change in the demand of good 1 into income and substitution effect.

9. Repeat the previous exercise, but instead of $p_1$ doubling, it is 4 times as high. So $p_1' = 4$.

10. Consider the utility function $u(x_1, x_2) = \ln(x_1) + x_2$. Initially $(p_1, p_2, m) = (1, 1, 4)$, but suddenly $p_1' = 2$. Decompose the total change in demand for good 1 into income and substitution effect.

11. In the interior solution of quasi-linear preferences, the hicksian and marshallian demands of good 1 are identical. Why is this the case?

**Answers**

1. I give you the expenditure functions. You can verify the rest.

   - $e(p_1, p_2, u) = \frac{4}{3}(3p_1)^{1/4}p_2^{3/4}u^{1/4}$
   - $e(p_1, p_2, u) = \min\left\{\frac{p_2}{3}, p_1\right\} \cdot u$
   - $e(p_1, p_2, u) = u \cdot \left(p_1 + \frac{p_2}{3}\right)$
   - $e(p_1, p_2, u) = \begin{cases} p_1 e^u & \text{if} \quad u \leq \ln(p_2) - \ln(p_1) \\ p_2 + p_2 u + p_2[\ln(p_1) - \ln(p_2)] & \text{if} \quad u > \ln(p_2) - \ln(p_1) \end{cases}$ and the hicksian for good 1 is: $h_1(p_1, p_2, u) = \begin{cases} \frac{p_2}{p_1} & \text{if} \quad u > \ln(p_2) - \ln(p_1) \\ e^u & \text{if} \quad u \leq \ln(p_2) - \ln(p_1) \end{cases}$

2.
   - $e(\cdot) = 2\sqrt{p_1 p_2 u}$, $h_1(\cdot) = \left(\frac{p_2}{p_1}u\right)^{1/2}$, and $h_2(\cdot) = \left(\frac{p_1}{p_2}u\right)^{1/2}$.
   - $e(\cdot) = \min\{p_1, p_2\} \cdot u$ and $h_1(\cdot) = \begin{cases} u & \text{if} \quad p_1 < p_2 \\ [0, u] & \text{if} \quad p_1 = p_2 \\ 0 & \text{if} \quad p_1 > p_2 \end{cases}$
   - $e(\cdot) = (p_1 + p_2)u$ and $h_1(\cdot) = h_2(\cdot) = u$.

3. At $(1, 1, 12)$ we have $x_1^* = 6$, $x_2^* = 6$, and $u^* = 36$.

   At $(3, 1, 12)$ we have $x_1^{**} = 2$, $x_2^{**} = 6$, and $u^{**} = 12$

   Calculate $h_1(p_1 = 3, p_2 = 1, u = 36) \simeq 3.46$

   Total effect on good 1 is $\text{TE} = 2 - 6 = -4$.

   Substitution effect on good 1 is $\text{SE} = 3.46 - 6 = -2.54$.

   Income effect is then $\text{IE} = -1.64$ because $\text{TE} = \text{SE} + \text{IE}$

4. Calculate $h_2(p_1 = 3, p_2 = 1, u = 36) \simeq 10.39$.

   Total effect on good 2 is $\text{TE} = 6 - 6 = 0$.

   Substitution effect on good 2 is $\text{SE} = 10.39 - 6 = 4.39$.

   Income effect is then $\text{IE} = -4.39$ because $\text{TE} = \text{SE} + \text{IE}$

5. See figure. Hicksian is the blue line. We can decompose into substitution and income effect as shown on the x axis.

6. At $(1, 1, 12)$ we have $x_1^* = 6$, $x_2^* = 6$, and $u^* = 6$.

   At $(3, 1, 12)$ we have $x_1^{**} = 3$, $x_2^{**} = 3$, and $u^{**} = 3$

   Calculate $h_1(p_1 = 3, p_2 = 1, u = 6) = 6$

   Total effect on good 1 is $\text{TE} = 3 - 6 = -3$.

   Substitution effect on good 1 is $\text{SE} = 6 - 6 = 0$.

   Income effect is then $\text{IE} = -3$ because $\text{TE} = \text{SE} + \text{IE}$

7. See figure. Because this is perfect complements, substitution effect is 0. Everything is income effect.

8. At $(1, 3, 12)$ we have $x_1^* = 12$, $x_2^* = 0$, and $u^* = 12$.

   At $(2, 3, 12)$ we have $x_1^{**} = 6$, $x_2^{**} = 0$, and $u^{**} = 6$

   Calculate $h_1(p_1 = 2, p_2 = 3, u = 12) = 12$

   Total effect on good 1 is $\text{TE} = 6 - 12 = -6$.

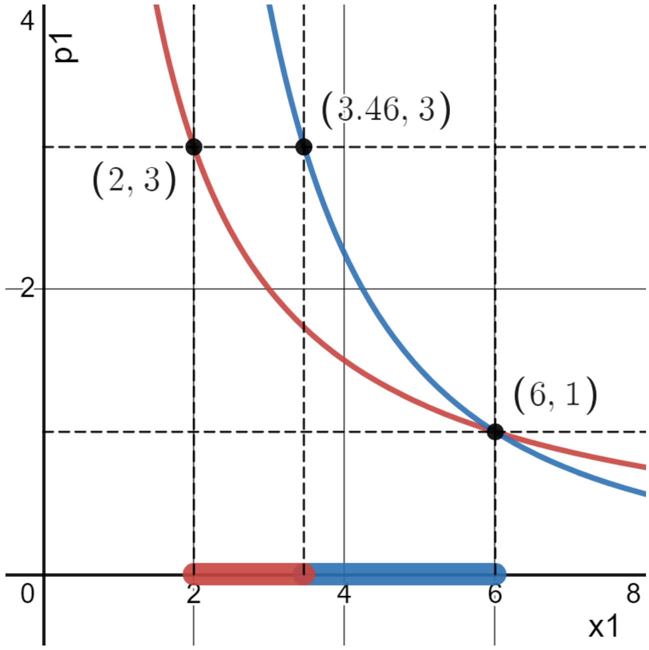


Figure 1: Exercise 5

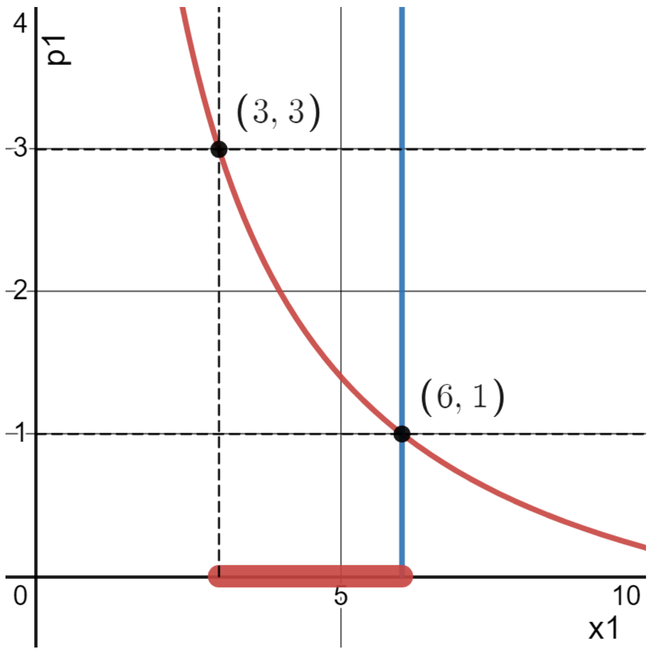


Figure 2: Exercise 7

Substitution effect on good 1 is SE $= 12 - 12 = 0$.

Income effect is then IE $= -6$ because TE $=$ SE $+$ IE

There is no substitution effect at all because good 1 is still the cheapest one.

9. At $(1, 3, 12)$ we have $x_1^* = 12$, $x_2^* = 0$, and $u^* = 12$.

   At $(4, 3, 12)$ we have $x_1^{**} = 0$, $x_2^{**} = 4$, and $u^{**} = 4$

   Calculate $h_1(p_1 = 4, p_2 = 3, u = 12) = 0$

   Total effect on good 1 is TE $= 0 - 12 = -12$.

   Substitution effect on good 1 is SE $= 0 - 12 = -12$.

   Income effect is then IE $= 0$ because TE $=$ SE $+$ IE

   Now that good 2 becomes cheaper, the full effect is all due to substitution.

10. At $(1, 1, 4)$ we have $x_1^* = 1$, $x_2^* = 3$, and $u^* = 3$.

    At $(2, 1, 4)$ we have $x_1^{**} = 1/2$, $x_2^{**} = 3$, and $u^{**} = \ln(1/2) + 3$

    Calculate $h_1(p_1 = 2, p_2 = 1, u = 3) = 1/2$

    Total effect on good 1 is TE $= 1/2 - 1 = -1/2$.

    Substitution effect on good 1 is SE $= 1/2 - 1 = -1/2$.

    Income effect is then IE $= 0$ because TE $=$ SE $+$ IE

    In the interior solution there is no income effect for good 1 because its marshallian demand does not depend on income.

11. Because good 1 does not have income effect in the interior solution.