Microeconomics
Homework 1: Preferences & the Budget Constraint

Javier Tasso

1. Given the budget constraint $p_1x_1 + p_1x_2 = m$, show what happens when:
   (a) p_1 falls.
   (b) p_2 falls.
   (c) p_1 and p_2 fall by the same proportion.
   (d) m increases.
   (e) (p_1, p_2, m) all fall/increase by the same proportion.
2. Plot the budget constraint $x_1 + 2x_2 = 12$. Suppose there's rationing and it's not possible to consume more than 10 units of good x_1. Plot the new budget constraint.
3. Suppose x_1 is bottles of water and x_2 represent other goods. Prices and income are $(p_1, p_2, m) = (2, 1, 12)$. Plot the budget constraint. Now suppose we give the consumer two bottles of water for free. Plot the new budget constraint.
4. Consider the budget constraint $x_1 + x_2 = 6$. Plot it. Now suppose that if you consume 3 or more units of good x_1, then you pay a 20% sales tax for unit 3 and above, making the price for those units 1.2. Plot the new budget constraint.
5. Consider the budget constraint $p_1x_1 + p_2x_2 = 12$. You don't know the prices (p_1, p_2), but you do know that one unit of x_1 trades with 3 units of good x_2. If the consumer spends all her money on x_1, she buys 4 units. Plot the budget constraint.
6. Argue graphically. If preferences are rational, the indifference curves cannot cross each other.
7. Argue graphically. If preferences are strongly monotone, the indifference curves are not fat.
8. Argue graphically. If preferences are strictly convex, the indifference curves are not fat.
9. Suppose a consumer does not care about good x_2, plot her indifference curves.
10. Suppose x_1 is a good and x_2 is a bad. Plot the indifference curves.
11. Suppose both x_1 and x_2 are bads. Plot the indifference curves.
12. Suppose $\succeq$ is complete and transitive:
    (a) Prove that $\succ$ is also transitive.
    (b) Prove that $\sim$ is also transitive.
13. Suppose $\succeq$ is complete and transitive:
    (a) Prove $\sim$ is reflexive.
    (b) Prove $\succ$ is not reflexive.
14. Prove that strong monotononicity implies weak monotonicity.

Answers

1. See figure. First panel is (a), second panel is (b), third panel is (c) and (d). The situation in (e) does not shift the budget constraint.

Figure 1: Exercise 1

2. See figure.

3. See figure.

4. See figure.

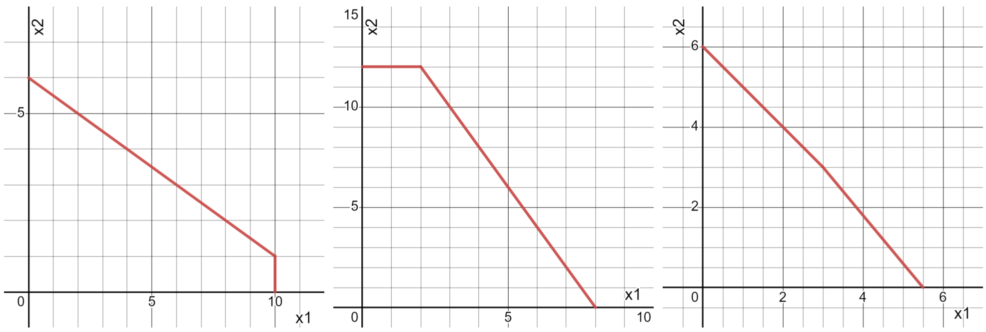

Figure 2: Exercise 2, 3, and 4

5. $p_1 = 3$ and $p_2 = 1$.

6. See figure. $A \succ C$ because it is in a higher indifference curve. At the same time $A \sim B$ and $B \sim C$, which implies $A \sim C$, a contradiction.

7. See figure. $A \sim B$ because they are in the same indifference curve. But A has more of both goods than B. By strong monotonicity we should have $A \succ B$, a contradiction.

8. See figure. $A \sim B$ because they are in the same indifference curve. $0.5A + 0.5B$ on the straight dashed line should be strictly preferred to both A and B because of strict convexity. But in the graph it is still indifferent, a contradiction.

9. The indifference curves are vertical and grow to the right.

10. The indifference curve have a positive slope and grow to the right.

11. The indifference curves look normal at first sight, but thew grow southwest instead of northeast.

12. (a) Proof that $\succ$ is also transitive.
    i. Suppose $x \succ y$ and $y \succ z$.
    ii. By definition, this implies $x \succeq y$ and $y \succeq z$. Transitivity of $\succeq$ here gives us $x \succeq z$.

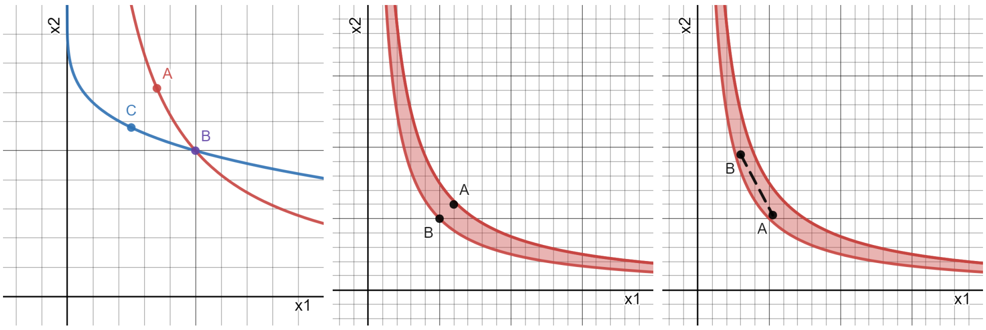

Figure 3: Exercise 6, 7, and 8

   iii. By definition, we also have $y \not\succeq x$ and $z \not\succeq y$.
   iv. Now suppose $z \succeq x$. By (ii) and transitivity of $\succeq$, we have $z \succeq y$. But this is a contradiction with (iii). So it must be that $z \not\succeq x$.
   v. This completes the proof. We have shown that $x \succeq z$ and $z \not\succeq x$, which means $x \succ z$.

(b) Proof that $\sim$ is also transitive.
   i. Suppose $x \sim y$ and $y \sim z$.
   ii. This means $x \succeq y$, $y \succeq x$, $y \succeq z$, and $z \succeq y$.
   iii. Transitivity of $\succeq$ also gives us that $x \succeq z$ and $z \succeq x$.
   iv. The last bulletpoint means $x \sim z$ and this completes the proof.

13. (a) Proof that $\sim$ is reflexive. Suppose it's not: $x \not\sim x$. This implies $x \not\succeq x$. But this is a contradiction with completeness of $\succeq$.
    (b) Proof that $\succ$ is not reflexive. Suppose it is: $x \succ x$. This implies $x \succeq x$, but at the same time $x \not\succeq x$ which is contradictory. So $\succ$ must not be reflexive.

14. M: if $x >> y$, then $x \succ y$. SM: if $x \geq y$ and $x \neq y$, then $x \succ y$. We want to show that SM $\implies$ M.
    i. Take bundles x and y such that $x >> y$.
    ii. Because x has more of every element than y, it is also true that $x \geq y$ and $x \neq y$.
    iii. By SM it must be that $x \succ y$ and this completes the proof.